

ON INDEPENDENCE DOMINATION

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Abstract. Let G be a graph. The independence-domination number $\gamma^i(G)$ is the maximum over all independent sets I in G of the minimal number of vertices needed to dominate I . In this paper we investigate the computational complexity of $\gamma^i(G)$ for graphs in several graph classes related to cographs. We present an exact exponential algorithm. We also present a PTAS for planar graphs.

1 Introduction

Let $G = (V, E)$ be a graph. A set A of vertices dominates a set B if

$$B \subseteq \bigcup_{x \in A} N[x].$$

The minimal cardinality of a set of vertices needed to dominate a set B is denoted by $\gamma_G(B)$. The domination number $\gamma(G)$ of the graph G is thus defined as $\gamma_G(V)$, where V is the set of vertices of G . When the graph G is clear from the context we omit the subscript G .

Definition 1. *The independence-domination number $\gamma^i(G)$ is*

$$\gamma^i(G) = \max \{ \gamma(A) \mid A \text{ is an independent set in } G \}.$$

Obviously, $\gamma(G) \geq \gamma^i(G)$. In [1] it was shown that $\gamma(G) = \gamma^i(G)$ for chordal graphs. Using this result Aharoni and Szabó showed that Vizing's conjecture on the domination number of the Cartesian product of graphs is true for chordal graphs, ie,

$$\gamma(G \square H) \geq \gamma(G) \cdot \gamma(H) \quad \text{when } G \text{ and } H \text{ are chordal [2].}$$

The Cartesian product $G \square H$ is the graph which has pairs (g, h) , $g \in V(G)$ and $h \in V(H)$ as its vertices. Two pairs (g_1, h_1) and (g_2, h_2) are adjacent in $G \square H$

if either $g_1 = g_2$ and $\{h_1, h_2\} \in E(H)$ or $\{g_1, g_2\} \in E(G)$ and $h_1 = h_2$ [34]. Vizing conjectured in 1968 [60] that

$$\text{for all graphs } G \text{ and } H \quad \gamma(G \square H) \geq \gamma(G) \cdot \gamma(H).$$

In 1994 Fisher proved that

$$\text{for all connected graphs } G \text{ and } H \quad \gamma(G \square H) \geq \gamma_f(G) \cdot \gamma(H), \quad (1)$$

where $\gamma_f(G)$ is the fractional domination number [25] (see also [12]). The fractional domination number is, by linear programming duality equal to the fractional 2-packing number (see, eg, [52]). For strongly chordal graphs $\gamma_f(G) = \gamma(G)$ [53] and, therefore, Vizing's conjecture is true for strongly chordal graphs. Recently, more progress was made by Suen and Tarr [55]. They proved that

$$\text{for all graphs } G \text{ and } H \quad \gamma(G \square H) \geq \frac{1}{2} \cdot \gamma(G) \cdot \gamma(H) + \frac{1}{2} \cdot \min \{ \gamma(G), \gamma(H) \}.$$

Actually, in [2] the authors show that for all graphs G and H

$$\gamma(G \square H) \geq \gamma^i(G) \cdot \gamma(H) \quad \text{and} \quad \gamma^i(G \square H) \geq \gamma^i(G) \cdot \gamma^i(H).$$

These result prompted us to investigate the computational complexity of $\gamma^i(G)$ for some classes of graphs. We find that especially cographs, and related classes of graphs, deserve interest since they are completely decomposable by joins and unions and they are therefore susceptible to proofs by induction. As far as we know, the computational complexity of $\gamma(G \square H)$ is still open for cographs.

Computing the domination number is NP-complete for chordal graphs [7,11], and this implies the NP-completeness for the independence domination. A similar proof as in [7] shows that independence domination is NP-complete for bipartite graphs. It is NP-complete to decide whether $\gamma^i(G) \geq 2$ for weakly chordal graphs [44]. The problem is polynomial for strongly chordal graphs [23].

2 Cographs

In this section we present our results for the class of cographs.

Definition 2. A *cograph* is a graph without induced P_4 .

Cographs are the graphs G that either have only one vertex, or for which either G or \bar{G} is disconnected [16]. Obviously, the class of graphs is hereditary in the induced subgraph order. It follows that a graph is a cograph if it is completely decomposable by joins and unions. We write $G = G_1 \oplus G_2$ when G is the union of two smaller cographs G_1 and G_2 and we write $G = G_1 \otimes G_2$ when G is the join of two smaller cographs G_1 and G_2 .

Theorem 1. *When G is a cograph with at least two vertices then*

$$\gamma(G) = \begin{cases} \min \{ \gamma(G_1), \gamma(G_2), 2 \} & \text{if } G = G_1 \otimes G_2, \\ \gamma(G_1) + \gamma(G_2) & \text{if } G = G_1 \oplus G_2. \end{cases}$$

Proof. When G is the union of two graphs G_1 and G_2 then $\gamma(G) = \gamma(G_1) + \gamma(G_2)$, since no vertex of G_1 dominates a vertex of G_2 and vice versa.

Assume that $G = G_1 \otimes G_2$. Any pair of vertex $x \in V(G_1)$ and $y \in V(G_2)$ is a dominating set. When one of G_1 or G_2 has a universal vertex then that is a universal vertex for G . This proves the formula for the join. \square

Remark 1. In [6] a graph G is called decomposable if its clique cover number is $\gamma(G)$, that is, if

$$\chi(\bar{G}) = \gamma(G).$$

The “A-class” is the collection of graphs that can be made decomposable by adding edges to it without changing the domination number. It is shown that graphs with domination number two, such as complete multi-partite graphs, belong to the A-class [6]. According to [6] Vizing’s conjecture holds true for graphs in A-class (see also [13, Theorem 2]). In [2] the authors raise the interesting question whether chordal graphs are A-class graphs.

Theorem 2. *Let G be a cograph. Then $\gamma^i(G)$ is the number of components of G .*

Proof. When G has only one vertex then $\gamma^i(G) = 1$.

Assume that $G = G_1 \otimes G_2$. Any maximal independent set is contained in G_1 or in G_2 . To dominate it, one needs only one vertex, from the other constituent.

Assume that $G = G_1 \oplus G_2$. Then any maximal independent set is the union of a maximal independent set in G_1 and G_2 . For the independence domination we have

$$\gamma^i(G) = \gamma^i(G_1) + \gamma^i(G_2).$$

By induction, $\gamma^i(G_j)$ is the number of components in G_j for $j \in \{1, 2\}$. \square

3 Distance-hereditary graphs

Distance-hereditary graphs were introduced by Howorka as those graphs in which for every pair of nonadjacent vertices all the chordless paths that connect them have the same length [32]. This class of graphs properly contains the class of cographs.

Distance-hereditary graphs G have a decomposition tree (T, f) which is described as follows (see [46] or, eg, [39]). Here, T is a rooted binary tree and f is

a bijection from the vertices of G to the leaves of T . Let e be an edge of T and let W_e be the set of vertices that are mapped to the leaves in the subtree rooted at e . The “twinset” $Q_e \subseteq W_e$ is the set of vertices that have neighbors in $V \setminus W_e$.

Each internal node p in the tree is labeled as \otimes or \oplus . Let e_1 and e_2 be the two edges that connect p with its children. Write Q_1 and Q_2 for the twinsets at e_1 and e_2 . If the label of p is \otimes then all vertices of Q_1 are adjacent to all vertices of Q_2 . If the label is \oplus then no vertex of Q_1 is adjacent to any vertex of Q_2 .

Let e be the edge that connects p with its parent. The twinset Q_e is either

$$Q_1 \text{ or } Q_2 \text{ or } Q_1 \cup Q_2 \text{ or } \emptyset.$$

The distance-hereditary graphs are exactly the graphs of rankwidth one. The decomposition tree above describes a rank-decomposition of width one.

Theorem 3. *There exists an $O(n^3)$ algorithm that computes the independence domination number for distance-hereditary graphs.*

Proof. The decomposition tree can be computed in linear time [19]. Let e be an edge in the decomposition tree. Let W_e be the set of vertices that are mapped to the leaves in the subtree and let Q_e be the twinset, ie, the set of vertices in W_e that have neighbors in $V \setminus W_e$.

The algorithm computes a table for each edge e in the decomposition tree. We write $H = G[W_e]$. For every pair of integers $a, g \in \{1, \dots, n\}$ the table stores a boolean value which is TRUE if there exists an independent set A in H with $|A| = a$ of which every vertex is dominated by a collection D vertices in H with $|D| = g$, except, possibly, some vertices in $A \cap Q_e$ (which are not dominated). The same table entry contains a boolean parameter which indicates whether there are vertices in $A \cap Q_e$ that are not dominated by the set D . A third boolean parameter indicates whether $D \cap Q_e$ is empty or not. Finally, a fourth boolean parameter stores whether some vertices of $D \cap Q_e$ dominate some vertices in $A \cap (W_e \setminus Q_e)$.

The information is conveniently stored in a symmetric 6×6 matrix. The rows and columns are partitioned according to the subsets

$$A, \quad D, \quad A \cap Q_e, \quad D \cap Q_e, \quad A \cap (W_e \setminus Q_e) \quad \text{and} \quad D \cap (W_e \setminus Q_e).$$

The diagonal entries indicate whether the subset is empty or not, and the off-diagonal entries indicate whether the subset of D either completely dominates all the vertices, or partly dominates some of the vertices, or does not dominate any vertex of the subset of A .

We describe shortly some cases that illustrate how a table for an edge e is computed. Consider a join operation at a node p . Let e_1 and e_2 be the two edges that connect p with its children. An independent set A in $G[W_e]$ can have vertices

only in one of the two twinsets Q_1 and Q_2 . Consider the case where $Q_e = Q_2$. When Q_1 has vertices in the independent set A which are not dominated by vertices in D_1 , then these vertices have to be dominated by a vertex from Q_2 . In case of a join operation, any (single) vertex of Q_2 can do the job. When a dominating set D_2 has vertices in Q_2 then this vertex dominates $A \cap Q_1$. Otherwise, a new vertex of Q_2 needs to be added to the dominating set.

It is easy to check that a table as described above can be computed for each edge e from similar tables stored at the two children of e . For brevity we omit further details. The independence number can be read from the table at the root. \square

Remark 2. It is easy to see that this generalizes to graphs of bounded rankwidth. As above, let (T, f) be a decomposition tree. Each edge e of T partitions the vertices of G into two sets. The cutmatrix of e is the submatrix of the adjacency matrix that has its rows indexed by the vertices in one part of the partition and its columns indexed by the vertices in the other part of the partition. A graph has rankwidth k if the rank over $\text{GF}[2]$ of every cutmatrix is at most k . For example, when G is distance hereditary, then every edge in the decomposition tree has a cutmatrix with a shape $\begin{pmatrix} J & 0 \\ 0 & 0 \end{pmatrix}$ where J is the all-ones matrix. Thus every cutmatrix has rank one. When a graph has bounded rankwidth then the twinset Q_e of every edge e has a partition into a bounded number of subsets. The vertices within each subset have the same neighbors in $V \setminus W_e$ [36]. A rank-decomposition tree of bounded width can be obtained in $O(n^3)$ time [46].

4 Permutation graphs

Another class of graphs that contains the cographs is the class of permutation graphs [27].

A permutation diagram consists of two horizontal lines in the plane and a collection of n line segments, each connecting a point on the topline with a point on the bottom line. A graph is a permutation graph if it is the intersection graph of the line segments in a permutation diagram.

In their paper Baker, Fishburn and Roberts characterize permutation graphs as follows [5]. (See also [21]; in this paper the authors characterize permutation graphs as interval containment graphs).

Theorem 4. *A graph G is a permutation graph if and only if G and \bar{G} are comparability graphs.*

Assume that G and \bar{G} are comparability graphs. Let F_1 and F_2 be transitive orientations of G and \bar{G} . A permutation diagram for G is obtained by ordering the vertices on the topline by the total order $F_1 \cup F_2$ and on the bottom line by the

total order $F_1^{-1} \cup F_2$. Permutation graphs can be recognized in linear time. The algorithm can be used to produce a permutation diagram in linear time [56].

Consider a permutation diagram for a permutation graph G . An independent set M in G corresponds with a collection of parallel line segments. The line segments of vertices in M are, therefore, linearly ordered, say left to right.

Definition 3. Consider a permutation diagram. An independent set M ends in x if the line segment of x is the right-most line segment of vertices in M .

Definition 4. For $x \in V$ and $k \in \mathbb{N}$, let $\mathcal{M}(x; k)$ be the collection of independent sets M that end in x and for which $\gamma(M) = k$.

Definition 5. Let $\Gamma(x; k)$ be the collection of minimum dominating sets for independent sets M that end in x with $\gamma(M) = k$.

The line segments of the neighbors of a vertex x are crossing the line segment of x . We say that z is a rightmost neighbor of x satisfying a certain condition, if the endpoint of z on either the topline or the bottom line is rightmost among all neighbors of x that satisfy the condition. Here, we allow that $z = x$.

Let $x \in V$ and let $z \in N[x]$. Define

$$\gamma_x(z) = \{ k \mid z \text{ is a right-most neighbor of } x \text{ and } z \in \Gamma \text{ for some } \Gamma \in \Gamma(x; k) \} \quad (2)$$

Lemma 1. Let G be a permutation graph and consider a permutation diagram for G . Then

$$\gamma^i(G) = \max \{ k \mid k \in \gamma_x(z) \quad x \in V \quad z \in N[x] \}. \quad (3)$$

Proof. Consider an independent set $M \subseteq V$ for which

$$\gamma(M) = \gamma^i(G).$$

Assume that M ends in x . Any set Γ that dominates M has a vertex $z \in N[x] \cap \Gamma$. Let z be a right-most neighbor of x which is in a dominating set Γ for M with $|\Gamma| = \gamma(M)$. Then

$$\gamma^i(G) = \gamma(M) \in \gamma_x(z). \quad (4)$$

This proves the lemma. \square

Theorem 5. There exists a polynomial-time algorithm that computes $\gamma^i(G)$ for permutation graphs.

Proof. We describe the algorithm to compute $\gamma_x(z)$. We assume that for every non-neighbor y of x that is to the left of x , the sets $\gamma_y(z')$ for $z' \in N[y]$ have been computed.

Consider an independent set $M \in \mathcal{M}(x; k)$. Let $z \in N[x]$ be a rightmost neighbor of x such that there is a dominating set $\Gamma \in \Gamma(x; k)$ with $z \in \Gamma$. Let $y \in M$ lie immediately to the left of x . When $z \in N(y)$ then z must be a rightmost neighbor of y . In that case

$$k \in \gamma_x(z) \iff k \in \gamma_y(z). \quad (5)$$

Now assume that $z \notin N(y)$. Then z dominates only one vertex of M , namely x . In that case z must be a right-most neighbor of x which is not in $N(y)$ and, if that is the case,

$$k \in \gamma_x(z) \iff \exists_{z' \in N[y] \setminus N(x)} k-1 \in \gamma_y(z'). \quad (6)$$

This proves the theorem. \square

5 Bounded treewidth

Graphs of bounded treewidth were introduced by Halin [29]. They play a major role in the research on graph minors [51]. Problems that can be formulated in monadic second-order logic can be solved in linear time for graphs of bounded treewidth. Graphs of bounded treewidth can be recognized in linear time [9,35]. Actually, bounded treewidth itself can be formulated in monadic second-order logic via a finite collection of forbidden minors [17].

Definition 6. Let $k \in \mathbb{N}$. A graph G has treewidth at most k if G is a subgraph of a chordal graph H with $\omega(H) \leq k+1$.

Theorem 6. Let $k \in \mathbb{N}$. There exists an $O(n^3)$ algorithm to compute $\gamma^i(G)$ when the treewidth of G is at most k .

Proof. Consider a tree-decomposition for G with bags of size at most $k+1$ [35,39]. Consider a subtree rooted at a node i . Denote the bag at node i by S_i . Denote the subgraph of G induced by the vertices that appear in bags in the subtree rooted at i by G_i . We use a technique similar to the one used in, eg, [57,58].

For all the subsets $A \subseteq S_i$, and for all pairs of integers p and q , let $b(p, q, A)$ denote a boolean value which is true if there exists an independent set M in G_i with p vertices with $M \cap S_i = A$. The vertices of A have a status, which is either white or gray. The white vertices of A are dominated by a set of q vertices in G_i and the gray vertices are not dominated by vertices in G_i .

It is easy to see that the boolean values can be computed in $O(n^2)$ time by dynamic programming for each node in the decomposition tree. \square

6 An exact exponential algorithm

In this section we describe an exact, exponential algorithm to compute the independence domination number [26,39].

Theorem 7. *There exists an $O^*(1.7972^n)$ algorithm to compute the independence domination number.*

Proof. Moon and Moser proved that a graph with n vertices has at most $3^{n/3}$ maximal independent sets [45]. Tsukiyama et al. showed that all the independent sets can be listed with polynomial delay [59].

First assume that there is a maximal independent set with at most $\beta \cdot n$ vertices. We determine the constant β later. Then $\gamma^i(G) \leq \gamma(G) \leq \beta \cdot n$.

For each maximal independent set M of size at most $\beta \cdot n$, we find the smallest set that dominates it as follows. Remove all edges except those that connect M and $V \setminus M$. Assume that every vertex of $V \setminus M$ has at most two neighbors in M . Then we can easily find $\gamma(M)$ in polynomial time via maximum matching. To see that, construct a graph H on the vertices of M where two vertices are adjacent if they have a common neighbor in $V \setminus M$. Let W be the set of vertices in M that are endpoints of edges in a maximum matching. Let $\nu(H)$ be the cardinality of a maximum matching in H . Then a solution is given by

$$\gamma(M) = \nu(H) + |M \setminus W|. \quad (7)$$

Otherwise, when at least some vertex of $V \setminus M$ has at least three neighbors in M , choose a vertex x of maximal degree at least three in $V \setminus M$ and branch as follows. In one branch the algorithm removes x and all its neighbors. In the other branch only the vertex x is removed. This gives a recurrence relation

$$T(n) \leq T(n-1) + T(n-4).$$

Since the depth of the search tree is bounded by $\beta \cdot n$, this part of the algorithm can be solved in $O^*(1.3803^{\beta \cdot n})$.

Assume that every maximal independent set has cardinality at least $\beta \cdot n$. In that case, we try all subsets of $V \setminus M$. The optimal value for β follows from the equation

$$1.3803^\beta = 2^{1-\beta} \quad \Rightarrow \quad \beta = 0.6827.$$

For the timebound we find that it is polynomially equivalent to

$$3^{n/3} \cdot 2^{(1-\beta)n} = 1.7972^n.$$

□

7 A PTAS for planar graphs

In this section we show that there is a polynomial-time approximation scheme for planar graphs. We use the well-known technique of Baker [4].

Consider a plane embedding of a planar graph G . Partition the vertices of G into layers L_1, \dots as follows. The outerface are the vertices of L_1 . Remove the vertices of L_1 . Then the new outerface are the vertices of L_2 . Continue this process until all vertices are in some layer.

If there are only k layers then the graph is called k -outerplanar.

Lemma 2 ([10]). *The treewidth of k -outerplanar graphs is at most $3k - 1$.*

Theorem 8. *Let G be a planar graph. For every $\epsilon > 0$ there exists a linear-time algorithm that computes an independence dominating set of cardinality at least*

$$(1 - \epsilon) \cdot \gamma^i(G).$$

Proof. Let $k \in \mathbb{N}$. Let $\ell \in \{1, \dots, k\}$ and consider removing layers

$$L_\ell, L_{\ell+k}, L_{\ell+2k}, \dots$$

Let $G(\ell, k)$ be the remaining graph. Then every component of G has at most k layers, and so $G(\ell, k)$ has treewidth at most $3k - 1$. Using the algorithm of Section 5 we can compute the independence domination numbers of $G(\ell, k)$, for $\ell \in \{1, \dots, k\}$.

Let M be an independent set in G with $\gamma(M) = \gamma^i(G)$. If we sum over $\ell \in \{1, \dots, k\}$, the vertices of M are counted $k - 1$ times. Each $\gamma^i(G(\ell, k))$ is at least as big as the dominating set that is needed to dominate the remaining vertices of M . Therefore, the sum over $\gamma^i(G(\ell, k))$ is at least $(k - 1) \cdot \gamma^i(G)$. Therefore, if we take the maximum of $\gamma^i(G(\ell, k))$ over $\ell \in \{1, \dots, k\}$ we find an approximation of size at least $(1 - \frac{1}{k}) \cdot \gamma^i(G)$. \square

8 Concluding remarks

One of our motivations to look into the independence domination number for classes of perfect graphs is the domination problem for edge-clique graphs of cographs. The main reason to look into this are the recent complexity results on edge-clique covers [18,33].

Let $G = (V, E)$ be a graph. The edge-clique graph $K_e(G)$ is the graph which has E as its vertices and in which two elements of E are adjacent when they are contained in a clique of G [3,15,28,49,50].

Let G and H be two graphs. The strong product $G \boxtimes H$ is the subgraph of $K_e(G \otimes H)$ induced by the edges that have one endpoint in G and the other in H . In other words, the vertices of $G \boxtimes H$ are pairs (g, h) with $g \in V(G)$ and

$h \in V(H)$. Two vertices (g_1, h_1) and (g_2, h_2) are adjacent when $g_1 \in N[g_2]$ and $h_1 \in N[h_2]$. It is well-known [40,41,48,54] that, when G and H are perfect,

$$\alpha(G \boxtimes H) = \alpha(G) \cdot \alpha(H).$$

Notice however that $G \boxtimes H$ itself is not necessarily perfect. For example $C_4 \boxtimes C_4$ contains an induced C_5 . The determination of $\alpha(G \boxtimes G)$ is very hard when G is not perfect. Lovász proved that $\alpha(C_5 \boxtimes C_5) = \sqrt{5}$ but, as far as we know, $\alpha(C_7 \boxtimes C_7)$ is open [41].

The independence number of the strong product has been investigated a lot due to its applications in data compression and coding theory. Very little is known about the (independent) domination number of strong products, although some investigations were made in [8,14,20,22,23,24,25,30,31,37,42,43,47,52,53].

As far as we know, the domination number for the edge-clique graphs of complete multipartite graphs is open. For simplicity, we call this the edge-domination number.³ A minimum edge-domination set is not necessarily realized by the complete bipartite subgraph induced by the two smallest color classes. For example, $K(2, 2, 2)$ has edge-domination number three while the complete bipartite $K(2, 2)$ has four edges. The edge-clique cover for complete multipartite graphs seems to be a very hard problem [38,42,47].

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³ One should be cautious because this terminology is also used for a different concept.

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